

On the logical structure of Bell theorems

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Bell theorems show how to experimentally falsify local realism. Conclusive falsification is highly desirable as it would provide support for the most profoundly counterintuitive feature of quantum theory—nonlocality. Despite the preponderance of evidence for quantum mechanics, practical limits on detector efficiency and the difficulty of coordinating space-like separated measurements have provided loopholes for a classical worldview; these loopholes have never been simultaneously closed. A number of new experiments have recently been proposed to close both loopholes at once. We show some of these novel designs fail in the most basic way, by not ruling out local hidden variable models, and we provide an explicit classical model to demonstrate this. They share a common flaw, which reveals a basic misunderstanding of how nonlocality proofs work. Given the time and resources now being devoted to such experiments, theoretical clarity is essential. Our explanation is presented in terms of simple logic and should serve to correct misconceptions and avoid future mistakes. We also show a nonlocality proof involving four participants which has interesting theoretical properties.

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INTRODUCTION

Some specific predictions of quantum mechanics are inconsistent with local realism [1]. Either these predictions are false or else our world is not locally realistic. These predictions can be tested, as quantum mechanics is a physical theory; however they are hard to verify indisputably. Independent measurements must be made upon systems that share an entangled quantum state yet which cannot possibly be dynamically linked, a state of affairs only achieved by space-like separation. Furthermore, these measurements must be sufficiently reliable to prevent any hypothetical local conspiracy from exploiting errors to create an illusory quantum effect. These two challenges, respectively the ‘locality loophole’ [2, 3] and the ‘detection loophole’ [4, 5], have yet to be met in a single experiment. To dispel this classical paranoia once and for all, we must find quantum predictions so profoundly divergent from local realism that the simultaneous closure of these loopholes is feasible.

A new kind of nonlocality proof has emerged in the recent literature and received widespread attention precisely for its apparent ability to close both loopholes simultaneously; examples are the two-photon experiments proposed by Cabello [14, 15] and by Greenberger, Horne and Zeilinger [16, 17]. These proofs simplify matters by reducing the number of different photon measurements required to violate classicality, thereby reducing the detector-efficiency threshold of the detection loophole to feasible levels. These proposals are flawed, however, in the sense that they do not rule out the most general type of local theory, exposing an important misconception concerning the structure of nonlocality proofs. The shortcut they take necessarily introduces an additional assumption into the proof procedure, and however plausible this assumption may be it allows local realism to evade contradiction. Though our argument is based on simple reasoning we are not just splitting logical hairs. This flaw allows local models to pass these ‘nonlocality tests’ with flying colours, as we show by explicit construction. Were such experiments performed with perfect detectors, they would still not falsify local realism. One of the two purposes of this paper is to explain clearly what this increasingly common mistake is, and how not to make it. A prior manuscript by two of the authors on the broader subject of “Entanglement swapping, light cones, and elements of reality” has presented some classical explanations of these proposed violations of local realism [18]. The quantum violations of local realism are one of the theory’s strangest and most perplexing features. If progress is to be made towards understanding our fundamentally

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nonclassical world, it is of paramount importance we understand precisely the experimental evidence in favour of nonlocality. This is especially important in light of the considerable resources now being devoted to realizing loophole-free experiments. There is much value therefore in a detailed examination of the structure of nonlocality proofs, and in exposing a tempting shortcut as a logical dead end.

The second purpose of this paper is to draw attention to an as-yet-unnoticed feature of Cabello's proposed experiment. Although his two-player proof is flawed, we show it can be converted into a valid four-player pseudo-telepathy game that comprises four nested three-party Mermin-GHZ games, played simultaneously upon the same quantum state. One might imagine that a state exhibiting four nested Mermin-GHZ type correlations would need each party to control more than one qubit. We show how to do this with each party holding just one qubit.

The paper is divided into three parts. In Section II we set the scene, introducing the key elements of nonlocality proofs, and recalling the salient features of Cabello's experimental proposal. Though our analysis is general we will focus upon one example for the sake of clarity, and we choose Cabello's design because it is perhaps the most convincing of its class, and has been clearly presented on a number of occasions in the literature [14, 15, 19]. This experimental model is set in a framework that has the potential to produce 'all-versus-nothing' violations of local realism: a non-maximally entangled four-qubit system. In Section III, we study this purportedly 'loophole-free' two-party Bell experiment and provide a classical model that perfectly reproduces all of the observed correlations, showing these proofs are not valid. We find the flaw lies in an unwarranted assumption about the nature of 'local elements of reality'. Interestingly, although such assumptions are intuitively reasonable, they are fatal to nonlocality proofs. We provide a detailed explanation of why this is so. Finally, in Section IV we show how a four-party analogue of the flawed proposal yields a valid pseudo-telepathy game and discuss its novel properties.

FOUR QUBIT NONLOCALITY

'Bell theorems without inequalities', also called 'nonlocality without inequalities experiments', are promising candidates for a loophole-free local realism falsification [6, 7, 8, 9]. They identify sets of measurements that, when performed independently upon an entangled system, produce a set of possible outcomes that is qualitatively different from any set of possible outcomes from any locally realistic model of the experiment. The discrepancies between the quantum and classical predictions come in two classes: (1) any locally realistic model will unavoidably produce outcomes that are never produced by the quantum system, and (2) no locally realistic model can produce all the outcomes that are produced by the quantum system [10]. Were such an experiment performed many times with perfect apparatus, the list of recorded outcomes would quickly convince us whether our experiment was behaving in a locally realistic fashion or not. Standard Bell-inequality experiments, in contrast, have no sharp distinction between the sets of outcomes; rather, it is the *frequency* of certain outcomes that is inexplicable by local hidden variables.

The first class of Bell theorems without inequalities, sometimes called 'all-versus-nothing' nonlocality proofs, are known to be equivalent to pseudo-telepathy games [10]. These provide an even stronger refutation of the local realistic viewpoint than the second class [10, 11]. Pseudo-telepathy games can be won all the time by players who share an entangled state, but players using any classical strategy instead will lose with a non-zero probability [12, 13]. The novel experiments designed to close both the locality and the detection loopholes all attempt to take the form of 'all-versus-nothing' nonlocality proofs.

Cabello presents two essentially identical nonlocality proofs in a four-qubit setting [14, 15]. It is well known that entangled four-qubit systems can provide violations of local realism, and this system is no exception. However, these four qubits are encoded onto a two-photon system. Here we concisely recall the ingredients.

We consider a four-qubit state prepared upon two photons entangled in both their polarization (H, V) and their path (u, d) degrees of freedom:

$$|\psi\rangle = \frac{1}{2}(|Hu\rangle_A|Hu\rangle_B + |Hd\rangle_A|Hd\rangle_B + |Vu\rangle_A|Vu\rangle_B - |Vd\rangle_A|Vd\rangle_B).$$

Rewriting this explicitly as a four-qubit state, we have:

$$|\psi\rangle = \frac{1}{2}(|0\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4 + |0\rangle_1|1\rangle_2|0\rangle_3|1\rangle_4 + |1\rangle_1|0\rangle_2|1\rangle_3|0\rangle_4 - |1\rangle_1|1\rangle_2|1\rangle_3|1\rangle_4). \quad (1)$$

Qubits 1 and 2 correspond to the polarization and path of Alice's photon respectively, and likewise for qubits 3 and 4 for Bob. Now consider the following three measurements X_j, Y_j and Z_j , performed individually on qubits j

($j = 1 \dots 4$):

$$\begin{aligned} X_j &= |0\rangle_j\langle 1| + |1\rangle_j\langle 0| \\ Y_j &= i(|1\rangle_j\langle 0| - |0\rangle_j\langle 1|) \\ Z_j &= |0\rangle_j\langle 0| - |1\rangle_j\langle 1|. \end{aligned} \quad (2)$$

Each of these measurements has two possible outcomes which we label $+1$ and -1 . Let the outcome of measurement X_j be written $x_j \in \{+1, -1\}$, and similarly for Y_j and Z_j . Quantum mechanics tells us that when appropriate measurements are made on state $|\psi\rangle$, the following fourteen equalities will always be found to hold:

$$z_1 = z_3, \quad (3)$$

$$z_2 = z_4, \quad (4)$$

$$x_1 = x_3 z_4, \quad (5)$$

$$x_2 = z_3 x_4, \quad (6)$$

$$x_1 z_2 = x_3, \quad (7)$$

$$z_1 x_2 = x_4, \quad (8)$$

$$y_1 = -y_3 z_4, \quad (9)$$

$$y_2 = -z_3 y_4, \quad (10)$$

$$y_1 z_2 = -y_3, \quad (11)$$

$$z_1 y_2 = -y_4, \quad (12)$$

$$x_1 x_2 = y_3 y_4, \quad (13)$$

$$x_1 y_2 = y_3 x_4, \quad (14)$$

$$y_1 x_2 = x_3 y_4 \quad \text{and} \quad (15)$$

$$y_1 y_2 = x_3 x_4. \quad (16)$$

There is no way to allot the values -1 and $+1$ to the twelve outcomes $\{x_j, y_j, z_j\}$ and satisfy all these equations simultaneously. A subset of just four equations, for instance (5),(9),(13) and (15) already leads to a contradiction. Therefore any physical theory that demands these values be preassigned before the measurement choices $\{X_j, Y_j, Z_j\}$ are made is not consistent with quantum mechanics.

This inconsistency can indeed be exploited to obtain an all-versus-nothing nonlocality proof. We must be careful, however, that the measurements $\{X_j, Y_j, Z_j\}$ are performed in such a way that local realism *requires* the values $\{x_j, y_j, z_j\}$ be preassigned. This is easy to guarantee if the four qubits are space-like separated, but a complication arises when the four qubit state $|\psi\rangle$ is instantiated upon Cabello's two-photon system. Qubits 1 and 2, the polarization and the path of Alice's photon, clearly cannot be measured at space-like separation. The same clearly applies to Bob's photon, so rather than making four independent qubit measurements chosen from three alternatives, we are really making two independent photon measurements chosen from nine alternatives:

$$\{(X_1, X_2); (X_1, Y_2); (X_1, Z_2); (Y_1, X_2); (Y_1, Y_2); (Y_1, Z_2); (Z_1, X_2); (Z_1, Y_2); (Z_1, Z_2)\}. \quad (17)$$

Cabello permits Alice and Bob to refrain from measuring one of their qubits, which leads to $9 + 6 = 15$ possible local measurements, but this complication does not affect the analysis. These two measurements each have four possible outcomes: $\{(-1, -1); (-1, +1); (+1, -1); (+1, +1)\}$. There is no *logical* reason to assume that just because $x_1 = 1$ when (X_1, X_2) is measured, x_1 would have equalled 1 if we had measured (X_1, Y_2) . Perhaps the different apparatus required to measure different path observables affects the photon's observed polarization? If we want to rule out this possibility, we must design our experiment very carefully. Quantum mechanics may tell us these measurements are independent, but nothing prevents local hidden variables from disobeying this rule!

THE LOGIC OF NONLOCALITY PROOFS

The 'nonlocality proof' of Section II works as follows. (Cabello's two papers provide two different descriptions of essentially the same proof; for ease of reference we discuss only that formulated in [14], but our objection and counterexample apply equally to the equivalent formulation in [15].) Alice randomly chooses to perform one of the following two measurements:

- 1a. X_1 and X_2 ?
- 2a. Y_1 and X_2 ?

Bob meanwhile randomly performs one of the following four measurements:

- 1b. X_3 and Y_4 ?
- 2b. X_3 and Z_4 ?
- 3b. Y_3 and Y_4 ?
- 4b. Y_3 and Z_4 ?

The only relevant equations are (5), (9), (13) and (15), as none of the other equations are ever tested by this experiment. Quantum mechanics predicts that these equations will always be satisfied. For this to be a valid nonlocality proof, there must be no way for a local hidden variable model to achieve the same thing. Yet the following classical model not only does exactly that, but also manages to perfectly mimic the quantum measurement statistics without even needing to hide the classical variables!

Let λ_1 , λ_2 and μ be three independent random bits taking the values $+1$ or -1 with equal probability. These will be the local hidden variables of our classical model. Instead of two entangled photons, Alice and Bob share a two-part system each part of which carries a copy of λ_1 , λ_2 ; Bob also has a copy of μ .

Alice's part of the system behaves as follows—regardless of whether she performs measurement 1a or 2a, it will simply output “ λ_1 and λ_2 ”:

- 1a. $\rightarrow \lambda_1$ and λ_2 .
- 2a. $\rightarrow \lambda_1$ and λ_2 .

Bob's system produces the following measurement outcomes:

- 1b. $\rightarrow \mu$ and $\mu\lambda_1\lambda_2$.
- 2b. $\rightarrow \mu$ and $\mu\lambda_1$.
- 3b. $\rightarrow \mu$ and $\mu\lambda_1\lambda_2$.
- 4b. $\rightarrow \mu$ and $-\mu\lambda_1$.

It is easy to see that in perfect agreement with quantum mechanics, the result of each individual ‘qubit’ measurement is completely random, yet the global correlations of equations (5), (9), (13) and (15) always hold. This local model is, in the context of this experiment, utterly indistinguishable from quantum mechanics itself, and this needs just two shared random bits and one private random bit to achieve. Since the experiment admits such a simple locally realistic explanation, it cannot have falsified local realism!

It is worth noting that while the above local model mimics quantum mechanics, this is not required if we simply want to pass the ‘nonlocality test’ set by the four equations (5), (9), (13) and (15). In this case an even simpler solution presents itself, whereby Alice and Bob needn't bother measuring their system at all, but simply respond in the following mechanical fashion:

Alice: always say “ $+1$ and $+1$ ”

Bob: if asked questions 1b, 2b or 3b, say “ $+1$ and $+1$ ”
if asked question 4b, say “ $+1$ and -1 ”.

These three lines suffice to prove that the experiment is not a nonlocality proof. For what would an experiment prove, when we know that the correlations it generated could be matched by two classical automata running this communication-free, postage-stamp sized protocol?

It is argued in [14, 15], that strategies such as this are forbidden in the new type of nonlocality proof. In the proposed model, Bob must always give the same answer to the question “What is z_4 ”, regardless of the context in which that question is asked: “*Since z_4 represents a local element of reality*, Bob's answer to Z_4 must be independent on whether Z_4 is asked together with Y_3 or X_3 ” (emphasis added). This is exactly the misconception at the heart of this and other recent proposals for ‘improved’ nonlocality proofs. We must not make any assumptions about what constitutes a local element of reality! Any alleged proof that spends any time whatsoever establishing ‘what the local elements of reality must be’ is likely to be wrong, or, at the very least, not as general as it should be.

Nonlocality proofs share a simple logical structure: they are proofs by contradiction. Two assumptions are made—the assumption of *locality* and the assumption of *realism*. A valid argument leads from these premises to a conclusion concerning the possible outcomes of measurements upon causally unconnected systems. It is then shown that this

conclusion is false if the predictions of quantum mechanics for certain space-like separated entangled states are true. When these predictions are experimentally verified, the conclusion is experimentally refuted, and therefore at least one of our two premises must have been false.

The new model for nonlocality proofs has a different, two-step structure, which Cabello erroneously attributes to Bell [19], and which is mirrored in the recent proposals by Greenberger, Horne and Zeilinger [16, 17]. In the first step, some predictions of quantum mechanics for the behaviour of a specific state $|\psi\rangle$ under a specific set of possible measurements $\{X_j, Y_j, Z_j\}$ are assumed to be true. To be specific, in the proposed model, it is concluded that pairs of measurements upon different qubits encoded on the same photon are outcome independent; the outcome of measurement A on qubit 1 is shown to be independent of the choice of measurement on qubit 2, and vice versa. A valid argument leads from this premise to a preliminary conclusion concerning the nature of viable local hidden-variable (LHV) models. In the second step, locality, realism, *and the conclusion of the first step* are assumed, and a deduction is made concerning the possible outcomes of measurements upon causally unconnected systems. It is then shown that this conclusion is false if some other predictions of quantum mechanics for the state $|\psi\rangle$ are true (to be specific, equations (5), (9), (13) and (15)).

The problem with this two-stage approach should be apparent. When we conduct the experiment presented in Section II using two photons, our logical conclusion will be shown to be inconsistent with observable evidence. We can deduce that at least one of the premises of our overall argument must have been false. However, the proposed new type of nonlocality proof has a total of three premises, not two! In addition to locality and realism, it is assumed from the outset that *in any LHV model, measurement outcomes that represent ‘local elements of reality’ (as defined by Einstein, Podolsky and Rosen) must be assigned definite values*. The proposed nonlocality proof does not test to see if this assumption is true for the system and measurements in question. Thus, the ensuing experiment will not rule out local realism. The third assumption can act as a ‘logical shield’, protecting locality and realism from contradiction. It must also be noted that the term coined by Cabello, ‘Einstein, Podolsky, Rosen local elements of reality’ or EPRLERs, is misleading. Einstein, Podolsky and Rosen never put forth a definition of a LER but only offered a criterion to *recognize* one [20]; they explicitly allowed for the possibility of other models.

There is nothing logically invalid about using three assumptions, instead of just the two. We certainly don’t reject the third premise because we’re forbidden from making spurious and unsupported assumptions about the properties of reality. After all, the assumptions of locality and realism are (surprisingly!) poorly physically motivated, whereas Cabello’s additional assumption is experimentally verifiable. At the end of the day, we can make any assumptions we want, but the conclusion we will end up drawing is that ‘one of our assumptions must be wrong’. If we want to rule out local realism, we’d better not have any additional assumptions in the way that can act as sacrificial pawns. If we have assumed some quantum predictions *without testing them*, logic dictates that these predictions might be wrong, however reasonable they seem. In this case, the application of logic may appear physically counterintuitive: an implicit assumption that quantum mechanics describes what is really physically happening leads to a proof with a classical solution! Nevertheless the logic is indisputable: the classical model is extremely simple and perfectly reproduces the supposedly nonlocal quantum correlations; an experiment with a classical explanation cannot prove nonclassicality. This highlights the value of proper logical analysis. The existence of an additional necessary assumption can be used as a test for the possibility of a local hidden variable solution, saving one the effort of exhaustively construction new local models of every specific case.

It is important to be very clear about our reasons for rejecting the additional assumption, so let us reiterate one last time. It is fatal to include an additional assumption in nonlocality proofs, even if that assumption is known to be *true* for quantum mechanics, because doing so can open the door to LHV models for which that additional assumption is *false*. Cabello’s errant assumption is surely true, as it is a mathematical property of quantum mechanics. Nevertheless, when the validity of quantum mechanics itself is at issue, it is a mistake to *assume* it and not *test* it, as must be clear from the simple counterexample presented above—quantum assumptions have led to a classical solution.

It is of course a physically observable fact that the measurement X_1 produces outcomes that are independent of the measurement performed on qubit 2, and it is quite possible to show that quantum mechanically it must be so. Why can we not perform this experiment separately, prove the errant assumption to our satisfaction, and then combine the results? Because doing so would introduce loopholes that dwarf those we are attempting to close. How could we claim to have closed the locality loophole, for instance, unless we performed the two experiments themselves at spacelike-separated locations? More damningly, how could we rule out a local physics that was sensitive to the differences between our experimental setups? Nonlocality proofs aim not just to meekly persuade us of the violation of local realism, but to logically compel us to accept it. This is their great strength. Confronted with the profoundly counterintuitive phenomenon of quantum nonlocality, it is vital to establish its reality as firmly as we can. Bell’s profound result was that we could falsify the entirety of local realism with one tantalisingly realizable experimental setup. But only if we do that experiment right!

How do we do things right? We must get rid of the additional assumption. We can redesign our experiment such that in parallel with everything else, it actually tests whether *all* the predicted behaviours of the quantum state $|\psi\rangle$ under measurements $\{X_j, Y_j, Z_j\}$ are observed, both equations (5),(9),(13),(15) and the independence of separate qubit measurements. This revision guarantees the only assumptions that might be false are locality and realism. Testing additional predictions means we will have to ask Alice and Bob to perform some additional measurements. It is exactly these measurements that Cabello adds to his original experiment in order to create a valid nonlocality proof in his recent response to criticism [19]. (Of course the validity of the extended experiment was never questioned, and does not imply the validity of the original smaller experiment, just as an attempt to test the Clauser, Horne, Shimony and Holt inequality that performed measurements in only one of the two nonorthogonal bases would not violate local realism. Half a valid proof is no proof at all.) However, the original proposal explicitly avoided testing these additional predictions in order to reduce the supposed maximum classical success rate to $\frac{3}{4}$ and thereby ease the burden placed on the photon detectors. As we have shown, this was unsuccessful. The valid extended experiment works because it tests all fourteen equations (3) to (16). A local hidden variable model can reproduce these correlations with probability at most $\frac{13}{14}$, significantly worse than other two-party proposals [21], and thus is not “stronger” or “loophole-free” in any meaningful sense.

There is a different way to make the original experiment valid. We can abandon the two-photon instantiation of $|\psi\rangle$ and consider four space-like separated qubits. The resulting experiment does not yield a better experimental proposal than the Mermin-GHZ pseudotelepathy game [7, 8] if we are concerned with closing the detection loophole or with minimizing the number of participants, but nevertheless has a number of interesting properties that we discuss in the next section.

THE FOUR-PARTY PSEUDO-TELEPATHY GAME

As mentioned in Section II, if the variables of equations (3)–(16) are to be taken as independent, then it is impossible to assign predetermined values to the variables in order to satisfy all these equations. An easy way to guarantee independence is to have all the measurements on the different qubits performed in space-like separated regions: in the language of pseudo-telepathy, give each qubit to different participants. Therefore, if we see the nonlocality argument based on equations (3)–(16) as a four party experimental setup, we do have a valid refutation of local realism.

It is interesting to mention that the subset of four equations, Equations (5), (9), (13) and (15), bears a striking resemblance to Mermin-GHZ’s equations in the three-party setting [7, 8, 12], where the second player is not ‘involved’. His output basically decides which Mermin-GHZ type game the other three participants are playing (see equations (18)–(25)). Furthermore, many different other combinations of equations will give the same scenario, with different participants deciding which game the others are playing. Since the Mermin-GHZ correlations are enough to refute the local realistic viewpoint, one might wonder why we should bother with the fourth player, the third measurement setting and the added number of different measurement setups of our scheme. From the perspective of closing the detection loophole, as far as we know, there is no justifiable reason. However, we would like to point out that this new pseudo-telepathy game does not require a GHZ state, but the $|\psi\rangle$ state. Therefore, under certain experimental situations, those where a GHZ state is difficult to create or manipulate, our scheme could be advantageous over the traditional Mermin-GHZ scheme.

We can see our game as four embedded three-player Mermin-GHZ games, each one having a different participant that is not involved. Picture the scenario as following: a referee designates one of the four players, say Bob, as a co-referee. Bob now has to choose whether Alice, Charlie or Didier will play the first Mermin-GHZ game:

$$x_1 = x_3 z_4, \quad (18)$$

$$y_1 = -y_3 z_4, \quad (19)$$

$$x_1 = y_3 y_4 \quad \text{and} \quad (20)$$

$$y_1 = x_3 y_4, \quad (21)$$

or whether they will play the second Mermin-GHZ game:

$$x_1 = x_3 z_4, \quad (22)$$

$$y_1 = -y_3 z_4, \quad (23)$$

$$x_1 = -y_3 y_4 \quad \text{and} \quad (24)$$

$$y_1 = -x_3 y_4. \quad (25)$$

It must be noted that the players do not need to know who was chosen as co-referee. Even the co-referee himself can be left in the dark concerning his role. They are simply asked questions which they must answer; the division lays in the head of the referee. It is also interesting to note that a similar game can be played with the players sharing a four-party GHZ state $(|0000\rangle + |1111\rangle)/\sqrt{2}$. However, it is surprising that this GHZ state is *not necessary*. Furthermore, one might be tempted to think that four three-party GHZ states are necessary, a total of twelve qubits, even if the game is not really four times the Mermin-GHZ game, while we can play this game with a low-dimensional state. It would also seem natural that Mermin-GHZ correlations could only arise with GHZ states. Our scheme demonstrates that it is not the case and that the $|\psi\rangle$ state is sufficient. This is rather surprising, because this state is not equivalent to the four-qubit GHZ state $(|0000\rangle + |1111\rangle)/\sqrt{2}$ under local unitary operations. It is interesting to point out that our game is reminiscent of the game presented in [22]. However, the game of [22] is presented as a game where players collude in order to form a standard Mermin-GHZ game.

CONCLUSION

Nonlocality is one of the most mysterious aspects of quantum mechanics and it is difficult to grasp for us living in an apparently classical world. Nonlocality proofs give us very useful tools to not only demonstrate nonlocality in theory, but also to implement convincing experiments in the laboratory. We have shown that a conceptual error in the design of nonlocality proofs can be fatal to the ultimate goal of such a proof, which is to demonstrate that our world is not locally realistic.

More precisely, we have elucidated why the description of a good nonlocality proof can (and should) be given *without any discussion of quantum mechanics or the nature of local elements of reality*. It is only the actual experimental setup, or the quantum winning strategy that needs to invoke quantum mechanics. We have shown that doing otherwise can fatally compromise the conclusions that can be drawn from nonlocality proofs. Because the two-participant nonlocality proofs of Cabello [14, 15] need to invoke properties of quantum mechanics as assumptions, we conclude that these proposals do not achieve their purported goal of ruling out locally realistic descriptions of our world, in spite of the fact that they do rule out some subclass of LHV models.

We also have shown that it is possible to modify Cabello's argument to make it a valid nonlocality proof by using a different method to that in [19]: by space-like separating the assumed-to-be-independent measurements, and thus enforcing the necessary measurement independence. This new construction leads to a pseudo-telepathy game with a most surprising feature. The proposed game shows that multiple Mermin-GHZ type correlations can be obtained from states with parties that control only low-dimensional non-maximally entangled systems.

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